

## Tuesday 15 January 2019

Morning (Time: 2 hours 30 minutes) $\quad$ Paper Reference 4MB1/02

## Mathematics B

## Paper 2

You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided - there may be more space than you need.
- Calculators may be used.


## Information

- The total mark for this paper is 100.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.


## Answer ALL TWELVE questions.

Write your answers in the spaces provided.
You must write down all the stages in your working.
1


Diagram NOT accurately drawn

## Figure 1

Figure 1 shows part of a regular 12 -sided polygon.
The vertices $B$ and $E$ of the polygon are joined with a straight line.
Calculate the size, in degrees, of $\angle A B E$.
Give reasons for each stage of your working.

2 Solve the simultaneous equations

$$
\begin{aligned}
3 x-2 y & =7 \\
x+6 y & =15
\end{aligned}
$$

Show clear algebraic working.

3 The original price of each 6-day ski pass is reduced by $15 \%$ in a sale.
In the sale the price of each 6-day ski pass is $\$ 272$
(a) Calculate the original price of each 6-day ski pass.

The price of each 3-day ski pass is $£ 110$
The exchange rate is $£ 1=\$ 1.70$
(b) Calculate how much Andrew will save by buying one 6-day ski pass in the sale rather than two 3-day ski passes.

4 (a) Express 56 as the product of its prime factors.

Trains to Watson leave Denby station every 56 minutes.
Trains to Barbe leave Denby station every 24 minutes.
A train to Watson and a train to Barbe both leave Denby station at 1200 .
(b) Find the next time that a train to Watson and a train to Barbe leave Denby station at the same time.

5 (a) On the grid opposite, draw the graph of $y=3 x+2$ for the values of $x$ from 0 to 5
(b) Show, by shading on the grid, the region $R$ defined by all of the inequalities

$$
y \leqslant 3 x+2 \text { and } y \geqslant 4 \text { and } 8 \leqslant 4 x \leqslant 18
$$

Label the region $R$.

Question 5 continued


6 One day, 80 customers at a health spa were asked if they used any of the gym $(G)$, the pool $(P)$ and the sauna ( $S$ ).

Of these 80 customers
34 had used the gym
60 had used the pool
30 had used the sauna
20 had used the gym and the pool but not the sauna
17 had used the pool and the sauna but not the gym
6 had used the gym, the pool and the sauna
no one had used the gym and the sauna but not the pool.
(a) Using this information, complete the Venn diagram to show the number of elements in each appropriate subset.

(b) Find (i) $\mathrm{n}\left([G \cup P \cup S]^{\prime}\right)$
(ii) $\mathrm{n}(G \cup S)$
(iii) $\mathrm{n}\left(P \cap S^{\prime}\right)$

Question 6 continued

7 Vincent plays a game in which he can score $0,1,2,3$ or 4 each time he plays the game. The score he gets in a game is independent of the score he got in the previous games. The table gives information about the probability of getting each score in a game.

| Score | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | $x$ | $3 x$ | $x$ | 0.2 | 0.1 |

Vincent plays the game 150 times.
(a) Calculate the number of times he would expect to score 1

Vincent plays the game another two times and records his two scores.
He adds these two scores together to get his Total.
(b) Calculate the probability that Vincent's Total is greater than 6

Given that for these two games Vincent has a Total greater than 6 points,
(c) calculate the probability that he got a score of 4 in the first of these two games.

Question 7 continued


Figure 2
Figure 2 shows the triangle $O A B$ with $\overrightarrow{O A}=2$ a and $\overrightarrow{O B}=3 \mathbf{b}$
The point $C$ lies on $O A$ such that $\overrightarrow{O C}=\frac{1}{3} \overrightarrow{O A}$
The point $D$ lies on $O B$ such that $\overrightarrow{O D}=\frac{2}{3} \overrightarrow{O B}$
(a) Find $\overrightarrow{C D}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

The point $P$ is such that $O D B P$ is a straight line and $A P$ is parallel to $C D$.
(b) Find $\overrightarrow{O P}$ in terms of $\mathbf{b}$.

The point $Q$ is such that $\overrightarrow{C D}=\overrightarrow{D Q}$
(c) Show that $A, B$ and $Q$ are collinear.

Question 8 continued

Question 8 continued

Question 8 continued

9 The vertices of triangle $A$ are the points with coordinates $(0,0),(1,0)$ and $(0,2)$
(a) On the grid opposite, draw and label triangle $A$.

Triangle $A$ is transformed to triangle $B$ under the transformation with matrix $\mathbf{M}$ where

$$
\mathbf{M}=\left(\begin{array}{rr}
3 & -4 \\
4 & 3
\end{array}\right)
$$

(b) On the grid, draw and label triangle $B$.

The transformation with matrix $\mathbf{M}$ is equivalent to an enlargement with centre the origin, with scale factor $k$ followed by an anticlockwise rotation of $\theta^{\circ}$ about the origin.
(c) Calculate the value of $k$.
(d) Calculate the value, to one decimal place, of $\theta$.

Triangle $B$ is transformed to triangle $C$ under a reflection in the $x$-axis.
(e) On the grid, draw and label triangle $C$.

Triangle $A$ is transformed to triangle $C$ under the transformation with matrix $\mathbf{T}$.
(f) Find matrix $\mathbf{T}$.

Question 9 continued


Turn over for a spare grid if you need to redraw your triangles.

Question 9 continued

## Question 9 continued

Only use this grid if you need to redraw your triangles.


10 A rocket is launched from a point which is 5 m above horizontal ground.
The rocket moves vertically upwards so that at time $t$ seconds, the height, $h$ metres, of the rocket above the ground is given by

$$
h=5+3 t+9 t^{2}-t^{3}
$$

At time $t$ seconds, the velocity of the rocket is $v \mathrm{~m} / \mathrm{s}$ and the acceleration of the rocket is $a \mathrm{~m} / \mathrm{s}^{2}$
(a) Find an expression for $v$ in terms of $t$.
(b) Find an expression for $a$ in terms of $t$.
(c) Find the time when the rocket stops accelerating upwards.

The rocket is instantaneously at rest when it is at point $A$.
(d) Show that the height, in metres to one decimal place, of $A$ above the ground is 131.2 m .

The rocket now falls vertically downwards and hits the ground.
(e) Find the total distance, to the nearest metre, travelled by the rocket at the instant it hits the ground.

$$
\left[\text { Solutions of } a x^{2}+b x+c=0 \text { are } x=\frac{-b \pm \sqrt{\left(b^{2}-4 a c\right)}}{2 a}\right]
$$

Question 10 continued
Quen

Question 10 continued

Question 10 continued

11

$$
f(x)=4 x^{3}-13 x-6
$$

(a) Use the factor theorem to show that $(2 x+1)$ is a factor of $\mathrm{f}(x)$.
(b) Hence factorise $\mathrm{f}(x)$ fully.

The curve $C$ has equation $y=\mathrm{f}(x)$
(c) Find the coordinates of the points of intersection of $C$ with the $x$-axis.
(d) Find the coordinates, to 2 decimal places, of the turning points of $C$.

The table below gives the coordinates of three points on $C$.

| $x$ | -2 | 0.5 | 1.5 |
| :---: | :---: | :---: | :---: |
| $y$ | -12 | -12 | -12 |

(e) On the grid opposite, draw the curve $C$ for $-2 \leqslant x \leqslant 2$

Clearly label the coordinates of the turning points of $C$ and the coordinates of the points of intersection with the $x$-axis and the $y$-axis.

Question 11 continued

$\qquad$

Turn over for a spare grid if you need to redraw your curve.

Question 11 continued

## Question 11 continued

Only use this grid if you need to redraw your curve.


12


Diagram NOT accurately drawn

Figure 3
Figure 3 shows a solid silver paperweight made from a cuboid and a half cylinder.
The cuboid is $2 a \mathrm{~cm}$ wide, $b \mathrm{~cm}$ long and $b \mathrm{~cm}$ high.
The plane face of the half cylinder coincides with the top face of the cuboid.
The total surface area of the paper weight is $A \mathrm{~cm}^{2}$
(a) Find an expression for $A$ in terms of $\pi, a$ and $b$.

Given that $a=6 \sqrt{5}$ and that the surface area of the paperweight can be written as

$$
\left(2 b^{2}+6 a b+60 \pi \sqrt{15}\right) \mathrm{cm}^{2}
$$

(b) show that the exact value of $b$ is $10 \sqrt{3}-6 \sqrt{5}$

The paperweight is melted down to form a different cuboid.
This second cuboid is $2 a \mathrm{~cm}$ wide, $b \mathrm{~cm}$ long and $h \mathrm{~cm}$ high, as shown in Figure 4.


Diagram NOT accurately drawn

Figure 4
(c) Calculate the size, to the nearest degree, of angle $G A C$.

Question 12 continued
Quen 12 cont
$\left[\begin{array}{cc}\text { Volume of cylinder } & \pi r^{2} h \\ \text { Curved surface area of cylinder } & 2 \pi r h\end{array}\right]$

Question 12 continued

Question 12 continued

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